Book of Abstracts

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COMPUTING STATIONARY SOLUTIONS OF PIECEWISE LINEAR SYSTEMS WITH PRIORITIES: APPLICATION TO EMERGENCY DEPARTMENTS

XAVIER ALLAMIGEON, PASCAL CAPETILLO, AND STÉPHANE GAUBERT

Medical emergency departments (EDs) are complex systems in which patients must be treated according to priority rules based on the severity of their condition. We develop models of EDs described by nonmonotone piecewise linear dynamical systems. These systems are closely related to the Bellman equations of finite horizon (semi-)Markov decision processes, with the difference that priority rules give rise to negative transition probabilities.

A central question is the existence of stationary (steady state) solutions—represented by invariant halflines—from which essential performance indicators, such as the throughput, can be derived. We prove that a broad class of these nonmonotone piecewise linear systems admits such stationary solutions. This result extends a fundamental theorem of Kohlberg (1980) on the existence of invariant halflines of nonexpansive piecewise linear maps. Our approach combines topological degree theory with Blackwell optimality.

We then develop a general method for computing the congestion phase diagram, which describes how stationary solutions vary in terms of the parameters representing system resources (such as senior doctors, interns, nurses, cubicles, etc., in the case of EDs). A key ingredient is a polynomial time algorithm to test whether a given policy (i.e., a set of bottleneck tasks) can be realized by some allocation of resources. This is done by a reduction to a feasibility problem for an unusual class of lexicographic polyhedra.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

GLOBAL STABILITY OF PERTURBED CHEMOSTAT SYSTEMS

CLAUDIA ÁLVAREZ-LATUZ, TÉRENCE BAYEN, AND JÉRÔME COVILLE

The chemostat system is a classic model for microbial evolution in controlled environments, famously predicting the Competitive Exclusion Principle (CEP) where only one species survives when competing for a single nutrient. Yet, real-world observations often defy this principle, showing coexistence of multiple species (though low-concentration). We demonstrate that this discrepancy can be explained by introducing a perturbation term into the system dynamics. Under biologically reasonable assumptions on the perturbation, we prove that the system remains globally asymptotically stable. Our main result is based on the Malkin-Gorshin Theorem and Smith and Waltman results on perturbed steady-states. After confirming stability, we apply numerical optimal control tools to optimize criteria such as production and biodiversity.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

PROJECTED SOLUTIONS OF QUASI-EQUILIBRIUM PROBLEMS AND GENERALIZED NASH EQUILIBRIUM PROBLEM: CONCEPT, RESULTS AND ITERATIVE METHODS

DIDIER AUSSEL AND JAUNY AND ASRIFA SULTANA AND SHIVANI VALESHA

A quasi-equilibrium problel is an equilibrium problem in which the constraint set depends on the considered point, and is thus a constraint map. An similar definition can be given for generalized Nash equilibrium problem. A classical assumption is that this constraints set-valued map is a self-map, that is the image set in included into the set on which the constraint map is defined. But when this constraint map is not a self-map, either because the image set is not included into the initial set or even they are not defined in the same space, then the concept of solution for such problems collapses since it intrinsically assumes the existence of fixed points for the constraint map. On the other hand, as shown in [1] for the case of bidding processes in electricity markets, considering such situations is very important in applications.

This is why the concept of *projected solutions* has been defined in [1]. We will here recall this concept, explore some recent generalizations and present two numerical schemes developed in [2] to compute such projected solutions.

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8-11 July 2025

REGULARIZATION OF OPTIMAL CONTROL PROBLEMS ON STRATIFIED DOMAINS USING ADDITIONAL CONTROLS

TÉRENCE BAYEN, ANAS BOUALI, AND ALAIN RAPAPORT

This presentation focuses on hybrid optimal control. The goal is to implement a new regularization technique. This technique has the advantage of relying on a weaker transversality assumption than the commonly used transversality conditions. For this purpose, we consider a Mayer optimal control problem governed by a dynamics defined regionally, *i.e.*, the state space is stratified into a family of disjoint regions with nonsmooth interfaces, and, in each region, the dynamics is given by a smooth expression:

$$\dot{x} = f_j(x,u), \quad \text{if } \varphi_j(x) < 0.$$

It is shown that this problem is equivalent to a new optimal control problem, with additional controls v_j taking values in $[0,1]$ and a (smooth) dynamics as a convex combination of the smooth dynamics $\sum_{j=1}^{N} v_j f_j(x,u_j)$, along with the following mixed control-state constraint:

$$(1-2v_j)\varphi_j(x) = |\varphi_j(x)|.$$

Next, we introduce a family of auxiliary optimal control problems. In these problems, we first regularize the nonsmooth interfaces. In addition, we consider the convex combination of smooth dynamics (only) within a boundary layer. Furthermore, we add a penalization term to the cost function to account for the mixed control-state constraint. Our main result is that solutions to these (smooth) problems converge (up to a subsequence) to a solution of the original one. It is obtained thanks to a new hypothesis related to solutions to the auxiliary problems, which is weaker than the transverse crossing condition of the literature.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

LYAPUNOV ANALYSIS FOR FISTA UNDER STRONG CONVEXITY

LUIS M. BRICEÑO-ARIAS

In this paper, we conduct a theoretical and numerical study of the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [1] under strong convexity assumptions. We propose an autonomous Lyapunov function that reflects the strong convexity of the objective function, whether it arises from the smooth or non-smooth component. This Lyapunov function decreases monotonically at a linear rate along the iterations of the algorithm for a fixed inertial parameter. Our analysis demonstrates that the best theoretical convergence guarantees for FISTA in this context are obtained when the full strong convexity is treated as if it belongs to the smooth part of the objective. Within this framework, we compare the performance of forward-backward splitting (FBS) and several FISTA variants, and find that this strategy leads FISTA to outperform all other configurations, including FBS. Moreover, we identify parameter regimes in which FBS yields better performance than FISTA when the strong convexity of the non-smooth part is not leveraged appropriately.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

LIPSCHITZ UPPER SEMICONTINUITY OF LINEAR INEQUALITY SYSTEMS

MARÍA J. CÁNOVAS

This talk is focused on the computation of the Lipschitz upper semicontinuity modulus of the feasible set mapping for linear inequality systems in the two parametric contexts of *right-hand side* (RHS, for short) and *full* perturbations; the respective results are traced out from [1] and [2]. The difference between both perturbation settings is emphasized. In particular, the polyhedral structure of the graph of the feasible set mapping under RHS perturbations enables us to apply classical results as those of Hoffman (1952) and Robinson (1981) to ensure the Lipschitz continuity of this mapping on its domain (which implies the Lipschitz upper semicontinuity at any element of its domain). In contrast, the graph of the feasible set mapping under full perturbations is no longer polyhedral (not even convex). This fact requires *ad hoc* techniques to analyze the Lipschitz upper semicontinuity property and its corresponding modulus. For completeness, and with the aim of pointing out the difference between both contexts, RHS *vs* full perturbations, the talk also discusses what happens with the so-called *Hoffman constant.*

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

A SIMPLER CHARACTERIZATION OF GBD

ANTONIN CHAMBOLLE

The space "GBD" (Generalized functions with Bounded Deformation) was introduced by G. Dal Maso around 2010 to study a variational approach to crack growth in linearized elasticity due to Francfort and Marigo [3]. The definition of this space is quite natural (when one obviously knows the energies introduced in [3]), but somewhat technical, and involves bounds on the derivatives in *all* directions of the space. It is known that for spaces of type "BD" (Bounded Deformation [5, 4]), one can characterize the membership of a function in the space by controlling its variation in a finite number of directions (d principal directions and $d-1$ directions combining the previous ones, in dimension $d \geq 2$, *cf* [1]). In this talk, we will try to explain how we demonstrate a new characterization of GBD based on the control of a finite number of directions. This is collaborative work with Vito Crismale, from the University of Rome La Sapienza [2].

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

DISSOCIATION LIMITS IN DENSITY FUNCTIONAL THEORY

GUY BOUCHITTÉ, GIUSEPPE BUTTAZZO, **THIERRY CHAMPION**, AND LUIGI DEPASCALE

In this talk we consider the Density Functional Theory (DFT) framework, where a functional of the form $F_\varepsilon(\rho) = \varepsilon T(\rho) + bC(\rho) - U(\rho)$ has to be minimized in the class of nonnegative measures ρ which have a prescribed total mass m (the total electronic charge). The parameter ε is small and the terms T, C, U respectively represent the kinetic energy, the electronic repulsive correlation, the potential interaction term between electrons and nuclei. Several expressions for the above terms have been considered in the literature and our framework is general enough to include most of them. It is known that in general, when the positive charge of the nuclei is small, the so-called *ionization phenomenon* may occur, consisting in the fact that the minimizers of F_ε can have a total mass lower than m; this physically means that some of the electrons may escape to infinity when the attraction of the nuclei is not strong enough. Our main goal in this talk is to illustrate the asymptotic behavior of the minimizers of F_ε as $\varepsilon \to 0$, , continuing the research we started in [1]. In particular we obtain that the Γ-limit functional is defined on sums of Dirac masses and has an explicit expression that depends on the terms T, C, U that the model takes into account. We shall illustrate this on some explicite examples.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

NEW CONVERGENCE RESULTS OF PROXIMAL ALGORITHMS IN THE PRESENCE OF ADJOINT MISMATCH

MARION SAVANIER, **EMILIE CHOUZENOUX**, JEAN-CHRISTOPHE PESQUET, AND CYRIL RIDDEL

We consider the proximal gradient algorithm (PGA) for solving penalized least-squares minimization problems arising in data science. This first-order algorithm is attractive due to its flexibility and minimal memory requirements allowing to tackle large-scale minimization problems involving non-smooth penalties. PGA can be accelerated by preconditioning strategies. A challenging situation arises when linear operators involved in this algorithm are replaced by approximations, with the aim to reduce the overall computation burden. In this talk, we study the PGA, in the two following important cases of model mismatch:

- Unmatched PGA, where the forward linear operator and its adjoint, involved in the least-square gradient step, are distinct [1];
- Unmatched preconditioned PGA, where two distinct preconditioning metrics are used in the gradient step and the proximity step [2].

For both new iterative scheme, we provide convergence conditions and characterize its limit point. Simulations in the context of X-ray tomographic image reconstruction from undersampled measurements show the benefits of our approach.

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STOCHASTIC KRASNOSEL'SKIĬ–MANN ITERATIONS

PATRICK COMBETTES AND JAVIER MADARIAGA

We propose a geometric framework to design a randomly relaxed and stochastically perturbed Krasnosel'skiĭ–Mann algorithm to construct a fixed point of an averaged nonexpansive operator acting on a Hilbert space. Our results extend on several fronts those of the existing literature and, in particular, [1, 2, 4, 5]. Several applications are discussed. This talk is based on [3].

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

MONOTONE RANDOMIZED APPORTIONMENT

JOSÉ CORREA

Apportionment is the act of distributing the seats of a legislature among political parties (or states) in proportion to their vote shares (or populations). A famous impossibility by Balinski and Young (2001) shows that no apportionment method can be proportional up to one seat (quota) while also responding monotonically to changes in the votes (population monotonicity). Grimmett (2004) proposed to overcome this impossibility by randomizing the apportionment, which can achieve quota as well as perfect proportionality and monotonicity – at least in terms of the expected number of seats awarded to each party. Still, the correlations between the seats awarded to different parties may exhibit bizarre non-monotonicities.

In this talk, we propose monotonicity axioms ruling out these paradoxes, and study which of them can be satisfied jointly with Grimmett's axioms. Essentially, we require that if a set of parties all receive more votes, the probability of those parties jointly receiving more seats should increase. Our work draws on a rich literature on unequal probability sampling in statistics (studied as dependent randomized rounding in computer science). Our main result shows that a sampling scheme due to Sampford (1967) satisfies Grimmett's axioms and a notion of higher-order correlation monotonicity.

This is a joint work with Paul Gölz, Ulrike Schmidt-Kraepelin, Jamie Tucker-Foltz, Victor Verdugo

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8-11 July 2025

DEGENERATION IN OPTIMAL CONTROL PROBLEMS WITH NON-REGULAR MIXED CONSTRAINTS

KARLA CORTEZ

In this talk, we explore the emergence of a phenomenon in the necessary conditions for optimal control problems with non-regular mixed constraints established in recent literature ([1], see also [2]). This effect mirrors the well-known degeneracy that arises in problems involving pure state constraints: in such situations, every admissible process formally satisfies the Pontryagin Maximum Principle (PMP), but only through the use of trivial yet nonzero multipliers, thus rendering the PMP ineffective for identifying potential minimizers.

We will analyze how the irregularity of the mixed constraints contributes to this degeneracy, leading to a loss of informativeness in the optimality conditions. To overcome this limitation, we introduce non-degeneracy criteria that guarantee the presence of meaningful, non-trivial multipliers. These theoretical insights will be supported by illustrative examples and preliminary results.

The talk will close with a brief outlook on potential avenues for extending the current framework and further validating the proposed conditions.

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NEW TIGHT BOUNDS FOR SGD WITHOUT VARIANCE ASSUMPTION: A COMPUTER-AIDED LYAPUNOV ANALYSIS

DANIEL CORTILD, **LUCAS KETELS**, JUAN PEYPOUQUET, AND GUILLAUME GARRIGOS

The analysis of Stochastic Gradient Descent (SGD) often relies on making some assumption on the variance of the stochastic gradients, which is usually not satisfied or difficult to verify in practice. This work [1] contributes to a recent line of works which attempt to provide guarantees without making any variance assumption, leveraging only the (strong) convexity and smoothness of the loss functions. In this context, we prove new theoretical bounds derived from the monotonicity of a simple Lyapunov energy, improving the current state-of-the-art and extending their validity to larger step-sizes. Our theoretical analysis is backed by a Performance Estimation Problem analysis [2], which allows us to claim that, empirically, the bias term in our bounds is tight within our framework.

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8-11 July 2025

CONDITIONAL INFIMUM, HIDDEN CONVEXITY AND THE S-PROCEDURE

JEAN-PHILIPPE CHANCELIER AND **MICHEL DE LARA**

Detecting hidden convexity is one of the tools to address nonconvex minimization problems and, possibly, to find global minimizers. We introduce the notion of conditional infimum, as it will prove instrumental in detecting hidden convexity. We develop the theory of the conditional infimum, and especially state a tower property, relevant for minimization problems. Thus equipped, we provide a sufficient condition for hidden convexity in nonconvex minimization problems. We illustrate our approach on two applications: we obtain new sufficient conditions i) to recast nonconvex quadratic minimization problems as convex ones ii) for the so-called S-procedure to hold true.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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QUADRATIC CONVEX REFORMULATION: BASIC CONCEPTS AND EXTENSIONS

SOUROUR ELLOUMI

We review the ideas and preliminary results of quadratic convex reformulation for finding exact (or global) solutions to quadratic programs with binary variables. Then, we show its extension to quadratic programs in mixed-integer variables. Lastly, we present an adaptation to polynomial problems in binary variables, as well as the links between this adaptation and the Lasserre hierarchy. We illustrate these concepts with numerical experiments.

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RELATIVELY MAS-COLELL PROPERNESS CONDITION AND QUASI RELATIVE INTERIOR

FABIÁN FLORES-BAZÁN

We strength the original notion of "properness" due to Mas-Colell and show its suitability for considering quasi relative interior of convex sets. This is employed to characterize when the Fenchel subdifferential of a (possibly nonconvex) function at a given point belonging to the quasi relative interior of the convex hull of its domain, is nonempty. As a consequence, we propose sufficient conditions related to quasiconvexity for the nonemptiness of such a subdifferential.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

SGD ET PAS DE POLYAK STOCHASTIQUE : ANALYSE D'UN ALGORITHME FRAUDULEUX MAIS PROMETTEUR

GOWER, GARRIGOS, LOIZOU, OIKONOMOU, MISCHENKO, SCHAIPP.

Le choix du pas de temps pour la mthode du gradient stochastique est crucial, que ce soit pour son analyse thorique ou lors d'une implmentation en pratique. En thorie il est souvent ncessaire de choisir le pas en fonction de paramtres du problme supposs connus (rgularit de la fonction, distance entre l'initialisation et la solution, etc). En pratique on choisit le pas parmi une liste de candidats sur une grille, ce qui est couteux en temps. Les mthodes dites *adaptives* cherchent se dbarasser de ces deux problmes, en fournissant des rgles pour choisir le pas qui dpendent le moins possible des proprits du problme.

C'est le cadre de cet expos: nous proposons d'tudier une version stochastique du *pas de Polyak*, qui est une formule propose par Polyak pour dfinir le pas en fonction des valeurs connues l'itr courant. Nous allons voir que ce choix s'avre trs puissant, permettant d'obtenir des vitesses de convergence *optimales* (en un certain sens) que ce soit dans le cadre lisse ou non lisse, le tout sans connaitre aucune des constantes classiques associes au problme.

Evidemment *il y a un hic*: pour pouvoir implmenter cet algorithme, l'utilisateur doit connaitre une nouvelle quantit aussi difficile atteindre que rsoudre le problme lui-mme! Nous discuterons alors de certains cas particuliers pour lesquels cette quantit est triviale (rgime dit de l'interpolation), ainsi que d'autres dans lequel elle peut tre raisonablement approche.

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DIFFERENTIALLY-PRIVATE SYNTHETIC DATA GENERATION: A SADDLE-POINT APPROACH

CRISTÓBAL GUZMÁN

Generating synthetic data is one of the key problems in private data analysis. In this talk, I will provide a summary of existing work, together with some novel approaches and results. In particular, we show that for accuracy guarantees based on relative error (i.e, a mixture of additive and multiplicative error), error rates can be made poly-logarithmic in the sample size, data universe size, and the number of linear queries; a result which is provably unattainable in the purely-additive counterpart.

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OPTIMALITY CONDITIONS AND SUBDIFFERENTIAL CALCULUS FOR INFINITE SUMS OF FUNCTIONS

ABDERRAHIM HANTOUTE

We provide some (uniform/firm/robust lower semicontinuity) decoupling techniques suitable for infinite optimization problems that are not necessarily convex. Consequently, fuzzy-like subdifferential calculus for sums of infinite collections of functions are established, along with necessary optimality conditions (multiplier rules) for uniform local minima. We do not rely on the Lipschitz continuity assumptions, which are usually involved in previous approaches. As an illustration of these results, specifically to motivate the use of infinite sums in optimization and variational analysis, we consider providing new dual schemes for general infinite convex optimization problems, where the corresponding dual is expressed as an infinite sum of the (weighted) constraints. Such duals allow shrinking the duality gap and guaranteeing a strong duality result.

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CLOSED-LOOP EQUILIBRIA FOR STACKELBERG GAMES: A STORY ABOUT STOCHASTIC TARGETS

CAMILO HERNÁNDEZ, **NICOLÁS HERNÁNDEZ**, EMMA HUBERT, AND DYLAN POSSAMAI

We provide a general approach to reformulating any continuous-time stochastic Stackelberg differential game under *closed-loop strategies* as a single-level optimisation problem with target constraints. More precisely, we consider a Stackelberg game in which the leader and the follower can both control the drift and the volatility of a stochastic output process, in order to maximise their respective expected utility. The aim is to characterise the Stackelberg equilibrium when the players adopt 'closed-loop strategies', *i.e.* their decisions are based *solely* on the historical information of the output process, excluding especially any direct dependence on the underlying driving noise, often unobservable in real-world applications. We first show that, by considering the—second-order—backward stochastic differential equation associated with the continuation utility of the follower as a controlled state variable for the leader, the latter's unconventional optimisation problem can be reformulated as a more standard stochastic control problem with target constraints. Thereafter, adapting the methodology developed by [1] or [2], the optimal strategies, as well as the corresponding value of the Stackelberg equilibrium, can be characterised through the solution of a well-specified system of Hamilton–Jacobi–Bellman equations. For a more comprehensive insight, we illustrate our approach through a simple example, facilitating both theoretical and numerical detailed comparisons with the solutions under different information structures studied in the literature.

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REACHING AN EQUILIBRIUM OF PRICES AND HOLDINGS OF GOODS THROUGH DIRECT BUYING AND SELLING

ALEJANDRO JOFRÉ

The Walras approach to equilibrium focuses on the existence of market prices at which, the total supplies match the total demands for goods. Trading activities that identify such prices by bringing agents together as potential buyers and sellers of a good are characteristically absent. Anyway, there is no money to pass from one to the other as ordinarily envisioned in buying and selling. Here, a different approach to equilibrium what it should mean and how it may be achieved is offered as a constructive alternative. Agents operate in an economic environment where adjustments to holdings have been necessary in the past and will likely be needed again in a future that is changing. Money is familiar for its role in facilitating these adjustments. Marginal utility provides relative values of goods for guidance in making incremental adjustments, and with cash incorporated into utility and taken as a numéraire, those values give money price thresholds at which an agent will be willing to buy or sell. Agents in pairs can then examine such individualized thresholds to determine whether a trade of some amount of a good for some amount of money may be mutually advantageous in leading to higher levels of utility. Iterative bilateral trades in this most basic sense, if they keep bringing all goods and agents into play, are guaranteed in the limit to reach an equilibrium state in which the agents all agree on prices and, under those prices, have no interest in further adjusting their holdings. The results of computer simulations are provided to illustrate how this process works.

Joint work with J. Deride (UAI) and R.T. Rockafellar (U. Washington).

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RECENT DEVELOPMENTS IN OPTIMAL CONTROL OF SWEEPING PROCESSES

LAILA ALSHARIEF AND **ABDERRAHIM JOURANI**

Our aim in this talk is to present recent development on optimal control of sweeping processes (OCSW). After having explored different approaches, we will show the fundamental rule that plays the coderivative of the metric projection mapping in obtaining a maximum principle of the implicit and explicit OCSW.

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GLOBAL OPTIMIZATION WITH HAMILTON-JACOBI PDES

MICHAEL HERTY, YUYANG HUANG, **DANTE KALISE**, AND NIKOLAS KANTAS

We introduce a novel approach to global optimization via continuous-time dynamic programming and Hamilton-Jacobi-Bellman (HJB) PDEs. For non-convex, non-smooth objective functions, we reformulate a global optimization problem as an infinite horizon, optimal asymptotic stabilization control problem. The solution to the associated HJB PDE provides a value function which corresponds to a (quasi)convexification of the original objective. Using the gradient of the value function, we obtain a feedback law driving any initial guess towards the global optimizer without requiring derivatives of the original objective. We then demonstrate that this HJB control law can be integrated into other global frameworks, such as consensus-based optimization, to improve its performance and robustness.

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LONG ORBIT OR EMPTY VALUE PRINCIPLE IN GENERALIZED METRIC SPACES

MILEN IVANOV, **DETELINA KAMBUROVA**, AND NADIA ZLATEVA

We explore generalizations of long orbit or empty value (LOEV) method introduced in [4] in more general spaces, where a distance function with certain properties is available, but not a metric. First, we lay the foundation in general topological spaces. Unlike in [4] no completeness of any kind is assumed. Then, we move towards completeness and more precisely sigma-semicompleteness as defined in [7]. We map the link to the correspondent Ekeland Variational Principle, Caristi's fixed point theorem and Takahashi's minimization principle and even characterize the space.

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LOT-SIZING AND PRODUCTION PLANNING FOR ENERGY APPLICATIONS

SAFIA KEDAD-SIDHOUM

The global transition towards renewable energy sources and the increasing complexity of energy management systems present important challenges in the field of energy optimization. Moreover, the increasing penetration of large-scale renewable generation capacity and the rising need for flexible demand-side management, such as smart battery charging and load shedding, have introduced new levels of complexity. These developments require simultaneous optimization of both energy production and flexible consumption. The talk will focus on production planning and lot-sizing problems in the context of energy applications. Starting from some fundamental results on simple lot-sizing problems, we will illustrate two recent applications in this field. The first focuses on energy management, with an emphasis on optimizing storage decisions. We propose mathematical formulations, highlight key complexity issues and discuss structural properties and polynomial algorithms[1]. The second application addresses fair electricity supply planning for collective self-consumption communities[2]. Specifically, we study communities with shared ownership and collective energy storage systems. We explore fairness criteria for the allocation of jointly produced renewable energy and shared resources.

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HEAVY-BALL AND NESTEROV ACCELERATIONS WITH HESSIAN DRIVEN DAMPING IN QUASICONVEX OPTIMIZATION

FELIPE LARA

Strongly quasiconvex functions were introduced by Polyak in 1966 [4] as the natural extension of the strongly convex functions. This class of functions, has been proved to be interesting for generalized convexity theory, since preserves both theoretical properties and linear convergence for first-order algorithms [1, 2]. In this talk, we focus on new properties for differentiable strongly quasiconvex functions via the behaviour of its gradient and we apply them to the study of second order dynamical system with Hessian driven damping [3]. Under mild assumptions, we provide exponential convergence of the trajectories to the optimal solution for the minimization problem of a differentiable strongly quasiconvex function. Furthermore, we study the discretization procedure for the heavy ball method and Nesterov accelerations, we provide linear convergence for both methods as well as mitigation of the oscillations commonly observed in traditional momentum methods. Finally, and if the time allow us, we present numerical illustrations.

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MEASURING THE STABILITY. A PARADIGMATIC PROBLEM IN OPTIMIZATION

MARCO A. LÓPEZ-CERDÁ

The increasing interest of the optimizers for stability analysis in the last decades obeys both to its beautiful theory and the appealing practical applications and algorithmic repercussions. In this talk we focus mainly on linear programming problems, and particularly on Lipschitz-type properties of the feasible set mapping, the optimal value function, and the optimal set (argmin) mapping. Roughly speaking, we aim to compute or estimate the rate of variation of feasible/optimal solutions with respect to the problem's data perturbations. Some of these properties are local (as Aubin property and calmness), as far as they concentrate around a certain solution nearby a given parameter. Some other properties (such as Hoffman stability) are of a global nature, since they tackle global variations of the whole solution set. We emphasize the fact that the quantitative stability measures provided in this talk are mainly point-based; thus they are conceptually implementable in practice.

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AN EXTENDED SPEED RESTART CRITERIA FOR A CONTINUOUS DYNAMICS WITH HESSIAN-DRIVEN DAMPING

HUIYUAN GUO, **JUAN JOSÉ MAULÉN**, AND JUAN PEYPOUQUET

Restart schemes are a popular technique for acceleration and stabilization of optimization algorithms. These routines stop the execution of the algorithm when some convergence rule is not satisfied, to restart the computations from a new starting point. Building on the continuous-time framework introduced in [1], we study a speed restart mechanism for the system

$$\ddot{x}(t) + \frac{\alpha}{t}\dot{x}(t) + \nabla\phi(x(t)) + \beta\nabla^2\phi(x(t))\dot{x}(t) = 0,$$

where $\alpha, \beta > 0$. In particular, we propose an extended speed-based restart scheme that generalizes previous approaches ([1, 2]) and prove a linear convergence rate for the objective function along the restarted trajectories. Numerical results demonstrate improved convergence for the continuous system and its discretizations.

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INERTIAL METHODS WITH VISCOUS AND HESSIAN DRIVEN DAMPING FOR NON-CONVEX OPTIMIZATION

RODRIGO MAULEN-SOTO, JALAL FADILI, AND PETER OCHS

In this work, we aim to study non-convex minimization problems via second-order (in-time) dynamics, including a non-vanishing viscous damping and a geometric Hessian-driven damping. Second-order systems that only rely on a viscous damping may suffer from oscillation problems towards the minima, while the inclusion of a Hessian-driven damping term is known to reduce this effect without explicit construction of the Hessian in practice. There are essentially two ways to introduce the Hessian-driven damping term: explicitly or implicitly. For each setting, we provide conditions on the damping coefficients to ensure convergence of the gradient towards zero. Moreover, if the objective function is definable, we show global convergence of the trajectory towards a critical point as well as convergence rates. Besides, in the autonomous case, if the objective function is Morse, we conclude that the trajectory converges to a local minimum of the objective for almost all initializations. We also study algorithmic schemes for both dynamics and prove all the previous properties in the discrete setting under proper choice of the step-size.

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OPTIMAL CONTROL OF FREE-TIME SWEEPING PROCESSES WITH APPLICATIONS

BORIS MORDUKHOVICH

This talk is devoted to a novel class of optimal control problems governed by the so-called sweeping (or Moreau) processes that are described by discontinuous dissipative differential inclusions with free time. Although such dynamical processes, strongly motivated by applications, have appeared in 1970s, optimal control problems for them have been formulated quite recently and occurred to be rather complicated from the viewpoint of developing control theory. Their study and applications require advanced tools of variational analysis and generalized differentiation, which will be presented in the lectures. Combining this machinery with the method of discrete approximations leads us deriving new necessary optimality conditions and their applications to practical models in elastoplasticity, traffic equilibria, robotics, etc.

Based on joint research with Giovanni Colombo (University of Padova), Dau Nguyen (San Diego State University), and Trang Nguyen (South Dakota State University).

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STOCHASTIC FACILITY LOCATION PROBLEM WITH OUTSOURCING COSTS

JAVIERA BARRERA, IVANA LJUBIC, AND **EDUARDO MORENO**

Stochastic facility location problems with outsourcing costs (SFLPOC) optimize facility placement and customer assignment under demand uncertainty. Excess demand beyond the capacity of a facility incurs outsourcing costs. This work addresses SFLPOC, aiming to minimize overall expected costs (placement, service and outsourcing). We model SFLPOC as a two-stage stochastic program. While prior work focused on specific assumptions or small scenario sets, we present methods suitable for general probability distributions. For discrete scenario sets, we improve upon classic Benders decomposition by exploiting the second-stage subproblem's structure. To handle general distributions, we partition the probability space based on incumbent integer solutions. Coupled with Benders cuts, this provides an exact solution method for common distributions (e.g. Bernoulli, Exponential, Poisson, Gaussian). Additionally, we introduce a compact formulation specifically for i.i.d. demand distributions, allowing us to solve even continuous distribution problems to optimality. Computational experiments on established benchmarks demonstrate that our compact formulation consistently finds optimal solutions, while the Benders approach provides strong solutions with proven optimality gaps for general distributions, outperforming sample average approximations.

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OPTIMIZING OVER PROBABILITY MEASURES TO MANAGE FLEXIBILITIES IN POWER SYSTEMS

NADIA OUDJANE

With the massive integration of renewable energies (photovoltaic (PV) and wind power) into the power grid, new uncertainties are impacting system balance. At the same time, advances in smart technologies and batteries offer the possibility of controlling the consumption of a large number of electrical appliances (electric vehicle recharging, heat pumps, etc.) which can contribute to system balance and thus compensate for the uncertainties induced by the integration of new renewable energies. In this framework, a major technical challenge is therefore to optimize the management of this large number of heterogeneous assets distributed across the network. This constitutes a large scale and nonconvex optimization problem which leads us to consider probabilistic relaxations where the decision variables are probability measures.

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RANDOMIZED BLOCK PROXIMAL METHOD WITH LOCALLY LIPSCHITZ CONTINUOUS GRADIENT

PEDRO PÉREZ-AROS

Block-coordinate algorithms are well recognized for their efficiency in tackling large-scale optimization problems, especially when computing full derivatives is memory-intensive and computationally expensive. In this talk, we present a randomized block proximal gradient algorithm for minimizing the sum of a differentiable function and a separable, proper lower semicontinuous function with both potentially nonconvex.

In contrast to existing methods, our approach only requires the partial gradients of the differentiable function to be locally Lipschitz continuous, eliminating the need for knowledge of global Lipschitz constants. A key feature of the algorithm is its ability to adaptively select stepsizes that ensure sufficient decrease, without prior information about local smoothness parameters.

Finally, the talk concludes with numerical experiments, including an image compression task using nonnegative matrix factorization, which illustrate the practical effectiveness of the proposed method.

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ACCELERATED GRADIENT METHODS VIA INERTIAL SYSTEMS WITH HESSIAN-DRIVEN DAMPING

JUAN PEYPOUQUET

We analyze the convergence rate of a family of inertial algorithms, which can be obtained by discretization of an inertial system with Hessian-driven damping. We recover a convergence rate, up to a factor of 2 speedup upon Nesterov's scheme, for smooth strongly convex functions. As a byproduct of our analyses, we also derive linear convergence rates for convex functions satisfying quadratic growth condition or Polyak-Łojasiewicz inequality. As a significant feature of our results, the dependence of the convergence rate on parameters of the inertial system/algorithm is revealed explicitly. This may help one get a better understanding of the acceleration mechanism underlying an inertial algorithm.

Based on joint work with Zepeng Wang, University of Groningen.

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SOLUTION OF MISMATCHED MONOTONE+LIPSCHITZ INCLUSION PROBLEMS

EMILIE CHOUZENOUX, **FERNANDO ROLDÁN**, AND JEAN-CHRISTOPHE PESQUET

Adjoint mismatch problems arises when the adjoint of a linear operator is replaced by an approximation, due to computational or physical issues. This occurs in inverse problems, particularly in computed tomography. In this talk we address the convergence of algorithms for solving monotone inclusions in real Hilbert spaces in the presence of adjoint mismatch. In particular, we investigate the case of a Lipschitz mismatch operator. We propose variants of the algorithms *Forward-Backward-Half-Forward* and *Forward-Half-Reflected-Backward* allowing to cope the mismatch. We establish conditions under the weak convergence to a solution of these variants is guaranteed. Moreover, the proposed algorithms allow each iteration to be implemented with a possibly iteration-dependent approximation to the mismatch operator, thus allowing this operator to be modified in each iteration.

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8-11 July 2025

EPICONVERGENCE IN ASPLUND WCG SPACES: ON ATTOUCH THEOREM.

RAFAEL CORREA, PEDRO PÉREZ-AROS, AND **JOSÉ P. SANTANDER**

We prove that a sequence of proper, convex, lower semicontinuous functions on a weakly compact generated Asplund space X epiconverges with the strong topology to a function f if and only if the sequence of the conjugates sequentially epiconverges with the w^* topology to the conjugate of f. Finally, we prove relations between epiconvergence and graphical convergence of subdifferentials in a mixed way, that is with respect to the product of strong and weak topology.

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8-11 July 2025

A NONSMOOTH EXTENSION OF THE BREZZI-RAPPAZ-RAVIART APPROXIMATION THEOREM VIA METRIC REGULARITY TECHNIQUES AND APPLICATIONS TO NONLINEAR PDES

FRANCISCO J. SILVA ÁLVAREZ

In this talk, based on a current collaboration with J. Berry and O. Ley (INSA Rennes), we discuss a general result allowing to obtain existence and a priori error estimates for approximations to nonlinear equations. This result is motivated by nonlinear partial differential equations and is a generalization a theorem of Brezzi, Rappaz and Raviart. In this context, our extension lies in the fact that the Nemytskii operator induced by the nonlinearity need not be differentiable. This is achieved by making use of the theory of metrically regular mappings. We also study a generalized differential for the Nemytskii operator. Finally we provide an application of our theorem to viscous Hamilton-Jacobi equations and second order mean field games.

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INSA Rouen Normandie, Rouen, France
8-11 July 2025

A LAGRANGIAN-BASED ADAPTIVE ALGORITHM FOR NONSMOOTH NONLINEAR COMPOSITE OPTIMIZATION

MARC TEBOULLE AND NADAV HALLAK

This talk introduces a novel adaptive augmented Lagrangian based framework for addressing a broad and challenging class of nonsmooth, nonconvex optimization problems with nonlinear composite objective structures. These problems characterized by the composition of nonsmooth, nonconvex functionals with nonlinear mappings are pervasive in modern applications. Yet, these models remain notoriously difficult to solve and fall outside the reach of most existing methods without imposing strong and restrictive structural assumptions.

Our approach introduces an adaptive mechanism to update feasibility-penalizing terms within the augmented Lagrangian framework. This innovation allows to essentially transform our multiplier type method into a simple and practical alternating minimization scheme from a certain iteration onward. Notably, this enables us to remove a key limitation in existing theory: the reliance on surjectivity type assumptions for convergence, an assumption traditionally required to establish convergence in composite settings. We show that our method converges to an ε-critical point with a well defined iteration complexity. Moreover, we prove that any bounded sequence generated by an inexact variant of the method with strictly decreasing tolerances, has limit points that are critical for the original problem. Our approach provides novel results even in the simpler *linear* composite models, in which the surjectivity requirement on the linear operator is a baseline assumption.

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CONTROLLABILITY PROPERTIES OF COUPLED PDES

EMMANUEL TRÉLAT

In a series of ongoing works with Hugo Lhachemi and Christophe Prieur, we investigate the controllability properties of some coupled PDEs, which can be of different natures.

For a heat-wave PDE with coupling at the boundary, we establish exact, exact null and approximate controllability in appropriate Hilbert spaces, under sharp assumptions. Our approach relies on an Ingham-Mnsz inequality, allowing us to establish an observability inequality for the dual problem. The resulting controllability space, which depends on the coupling function and is characterized in a spectral way, is not a conventional functional space.

If time allows, I will also give some results for other cascade systems, providing new results for coupled heat equations where interesting changes happen at some specific times in the controllability properties.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

OPTIMAL CONTROL IN WASSERSTEIN SPACES WITH APPLICATIONS TO NEURAL ODES

FERNANDA URREA, HASNAA ZIDANI, AND JALAL FADILI

The presentation will begin with a brief overview of necessary optimality conditions for control problems in Wasserstein spaces. This involves the development of for a Bolza optimal control problem governed by a non-local continuity equation. In the second part, we investigate a specific instance of this setting that highlights a strong connection with neural ordinary differential equations (neural ODEs). In this model, the control variable plays the role of network parameters, and the evolution of probability measures mirrors the forward pass of a neural network. Numerical examples will be presented to illustrate the performance and stabilization process of the approach in some scenarios.

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DIFFERENTIABILITY OF PROBABILITY FUNCTIONS INVOLVING STAR-SHAPED VALUED SET-VALUED MAPS.

WIM VAN ACKOOIJ, PEDRO PÉREZ-AROS, AND CLAUDIA SOTO

Motivated by a control-theoretic example in which system dynamics are influenced by uncertainty and a predetermined "here-and-now" control input, we define a feasible trajectory as one that reaches the target with high probability. This leads to a probability function that quantifies the likelihood associated with a random vector belonging to a star-shaped set. Consequently, we are motivated to explore the properties of probability functions acting on parameter-dependent star-shaped sets.

In this talk, we illustrate through examples that, although star-shapedness closely resembles convexity, it is not sufficient to guarantee locally Lipschitz continuity of the associated probability functions. To address this limitation, we propose an intuitive regularization of the star-shapedness concept. Under this regularization, and with additional reasonable assumptions, we derive "computable" expressions for the generalized subdifferentials of the resulting probability functions.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

A COST-BASED APPROACH TO SWEEPING PROCESSES

FRANCISCO VENEGAS

Moreau's sweeping process is a well known first-order differential inclusion, involving the normal cone to a time-dependent moving set. This differential inclusion can be solved using the *catching-up algorithm*, an iterative scheme based on successive projections. In this work, we propose a modified version of Moreau's catching-up algorithm, where the projection step is replaced by the minimization of a prescribed *cost function* $c : H \times H \to [0, \infty)$. In other words, this new algorithm yields sequences of points $(x_i)_{i=0}^N$ satisfying the state constraint, and such that x_{i+1} minimizes the cost $c(x_i, \cdot)$. We show that under reasonable assumptions on the cost function c, the sequences given by the algorithm converge to a trajectory $x(t)$ satisfying a differential inclusion of the form

$$Q(x(t))\dot{x}(t) \in -N_{K(t)}(x(t)) \quad \text{a.e in } [0, T],$$

where $Q : H \to \mathcal{B}(H)$ is determined by the cost function c. This cost-based approach offers a flexible modeling framework, allowing the cost function to promote or penalize specific trajectory behaviors. This talk is based on a joint work Emilio Vilches.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

NEW COMBINATORIAL INSIGHTS FOR MONOTONE APPORTIONMENT

VÍCTOR VERDUGO

The *apportionment problem* constitutes a fundamental problem in democratic societies: How to distribute a fixed number of seats among a set of states in proportion to the states' populations? This—seemingly simple—task has led to a rich literature and has become well known in the context of the US House of Representatives. We first focus on the well-studied family of *stationary divisor methods*, which satisfy the strong *population monotonicity* property, and show that this family produces only a slightly superlinear number of different outputs as a function of the number of states. While our upper and lower bounds leave a small gap, we show that—surprisingly—closing this gap would solve a long-standing open problem from discrete geometry, known as the complexity of k-levels in line arrangements. The main downside of divisor methods is their violation of the *quota* axiom, i.e., every state should receive $\lfloor q_i \rfloor$ or $\lceil q_i \rceil$ seats, where q_i is the proportional share of the state. As we show that randomizing over divisor methods can only partially overcome this issue, we propose a relaxed version of divisor methods in which the total number of seats may slightly deviate from the house size. By randomizing over these methods, we can simultaneously satisfy population monotonicity, quota, and ex-ante proportionality. Finally, we turn our attention to quota-compliant methods that are *house-monotone*, i.e., no state may lose a seat when the house size is increased. We provide a polyhedral characterization based on network flows, which implies a simple description of all ex-ante proportional randomized methods that are house-monotone and quota-compliant.

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Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

NEW INSIGHTS INTO MOREAU'S SWEEPING PROCESSES

EMILIO VILCHES

The sweeping process is a first-order differential inclusion involving the normal cone to a family of moving sets. It was introduced by J.J. Moreau in the early seventies to address an elastoplastic problem. Since then, it has been used to model constrained dynamical systems, nonsmooth electrical circuits, crowd motion, mechanical problems, and other applications.

The aim of this talk is twofold. On the one hand, we will provide the main insights into the well-posedness of the sweeping process, and on the other hand, we will present the latest developments in the subject, such as optimal control and numerical approximation.

This talk is based on the papers [1, 2, 3].

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Dixièmes Journées Franco-Chiliennes d'Optimisation
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8-11 July 2025

A PRIMAL APPROACH TO THE CLARKE-LEDYAEV INEQUALITY

MIHAIL HAMAMDJIEV, MILEN IVANOV, AND **NADIA ZLATEVA**

We present a version of the Clarke-Ledyaev inequality that does not involve elements of the dual space. The proof relies mainly on geometry and on the classical lemma of Bishop and Phelps. In addition, this approach allows us to provide a simplified proof of the Clakre-Ledyaev inequality. The approach is primal in the sense that no dual arguments are used. The results are published in [1, 2].

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