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COMPUTING STATIONARY SOLUTIONS OF PIECEWISE LINEAR SYSTEMS WITH PRIORITIES: APPLICATION TO EMERGENCY DEPARTMENTS

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Medical emergency departments (EDs) are complex systems in which patients must be treated according to priority rules based on the severity of their condition. We develop models of EDs described by nonmonotone piecewise linear dynamical systems. These systems are closely related to the Bellman equations of finite horizon (semi-)Markov decision processes, with the difference that priority rules give rise to negative transition probabilities.

A central question is the existence of stationary (steady state) solutions—represented by invariant halflines—from which essential performance indicators, such as the throughput, can be derived. We prove that a broad class of these nonmonotone piecewise linear systems admits such stationary solutions. This result extends a fundamental theorem of Kohlberg (1980) on the existence of invariant halflines of nonexpansive piecewise linear maps. Our approach combines topological degree theory with Blackwell optimality.

We then develop a general method for computing the congestion phase diagram, which describes how stationary solutions vary in terms of the parameters representing system resources (such as senior doctors, interns, nurses, cubicles, etc., in the case of EDs). A key ingredient is a polynomial time algorithm to test whether a given policy (i.e., a set of bottleneck tasks) can be realized by some allocation of resources. This is done by a reduction to a feasibility problem for an unusual class of lexicographic polyhedra.

References

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