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GLOBAL STABILITY OF PERTURBED CHEMOSTAT SYSTEMS

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The chemostat system is a classic model for microbial evolution in controlled environments, famously predicting the Competitive Exclusion Principle (CEP) where only one species survives when competing for a single nutrient. Yet, real-world observations often defy this principle, showing coexistence of multiple species (though low-concentration). We demonstrate that this discrepancy can be explained by introducing a perturbation term into the system dynamics. Under biologically reasonable assumptions on the perturbation, we prove that the system remains globally asymptotically stable. Our main result is based on the Malkin-Gorshin Theorem and Smith and Waltman results on perturbed steady-states. After confirming stability, we apply numerical optimal control tools to optimize criteria such as production and biodiversity.

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