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MONOTONE RANDOMIZED APPORTIONMENT

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Apportionment is the act of distributing the seats of a legislature among political parties (or states) in proportion to their vote shares (or populations). A famous impossibility by Balinski and Young (2001) shows that no apportionment method can be proportional up to one seat (quota) while also responding monotonically to changes in the votes (population monotonicity). Grimmett (2004) proposed to overcome this impossibility by randomizing the apportionment, which can achieve quota as well as perfect proportionality and monotonicity – at least in terms of the expected number of seats awarded to each party. Still, the correlations between the seats awarded to different parties may exhibit bizarre non-monotonicities.

In this talk, we propose monotonicity axioms ruling out these paradoxes, and study which of them can be satisfied jointly with Grimmett’s axioms. Essentially, we require that if a set of parties all receive more votes, the probability of those parties jointly receiving more seats should increase. Our work draws on a rich literature on unequal probability sampling in statistics (studied as dependent randomized rounding in computer science). Our main result shows that a sampling scheme due to Sampford (1967) satisfies Grimmett’s axioms and a notion of higher-order correlation monotonicity.

This is a joint work with Paul Gözl, Ulrike Schmidt-Kraepelin, Jamie Tucker-Foltz, Victor Verdugo