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CONDITIONAL INFIMUM, HIDDEN CONVEXITY
AND THE S-PROCEDURE

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Detecting hidden convexity is one of the tools to address nonconvex minimization problems and, possibly, to find global minimizers. We introduce the notion of conditional infimum, as it will prove instrumental in detecting hidden convexity. We develop the theory of the conditional infimum, and especially state a tower property, relevant for minimization problems. Thus equipped, we provide a sufficient condition for hidden convexity in nonconvex minimization problems. We illustrate our approach on two applications: we obtain new sufficient conditions i) to recast nonconvex quadratic minimization problems as convex ones ii) for the so-called S-procedure to hold true.

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