Dixièmes Journées Franco-Chiliennes d'Optimisation
INSA Rouen Normandie, Rouen, France
8-11 July 2025

CLOSED-LOOP EQUILIBRIA FOR STACKELBERG GAMES: A STORY ABOUT STOCHASTIC TARGETS

CAMILO HERNÁNDEZ, **NICOLÁS HERNÁNDEZ**, EMMA HUBERT, AND DYLAN POSSAMAI

We provide a general approach to reformulating any continuous-time stochastic Stackelberg differential game under *closed-loop strategies* as a single-level optimisation problem with target constraints. More precisely, we consider a Stackelberg game in which the leader and the follower can both control the drift and the volatility of a stochastic output process, in order to maximise their respective expected utility. The aim is to characterise the Stackelberg equilibrium when the players adopt 'closed-loop strategies', *i.e.* their decisions are based *solely* on the historical information of the output process, excluding especially any direct dependence on the underlying driving noise, often unobservable in real-world applications. We first show that, by considering the—second-order—backward stochastic differential equation associated with the continuation utility of the follower as a controlled state variable for the leader, the latter's unconventional optimisation problem can be reformulated as a more standard stochastic control problem with target constraints. Thereafter, adapting the methodology developed by [1] or [2], the optimal strategies, as well as the corresponding value of the Stackelberg equilibrium, can be characterised through the solution of a well-specified system of Hamilton–Jacobi–Bellman equations. For a more comprehensive insight, we illustrate our approach through a simple example, facilitating both theoretical and numerical detailed comparisons with the solutions under different information structures studied in the literature.

References

[1] H.M. Soner and N. Touzi: *Dynamic programming for stochastic target problems and geometric flows*, Journal of the European Mathematical Society, 3, 201–236, 2002.

[2] B. Bouchard, R. Élie and C. Imbert: *Optimal control under stochastic target constraints*, SIAM Journal on Control and Optimization, 5, 3501–3531, 2010.

Departamento de Matemática, Universidad Técnica Federico Santa María, Chile, email: nicolas.hernandezs@usm.cl.

This work is partially funded by ANID FONDECYT Iniciación 11240944.