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**LONG ORBIT OR EMPTY VALUE PRINCIPLE IN GENERALIZED
METRIC SPACES**

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We explore generalizations of long orbit or empty value (LOEV) method introduced in [4] in more general spaces, where a distance function with certain properties is available, but not a metric. First, we lay the foundation in general topological spaces. Unlike in [4] no completeness of any kind is assumed. Then, we move towards completeness and more precisely sigma-semicompleteness as defined in [7]. We map the link to the correspondent Ekeland Variational Principle, Caristi’s fixed point theorem and Takahashi’s minimization principle and even characterize the space.

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